NOISE IN IMAGING CHAINS: CORRELATIONS AND PREDICTIONS

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ABSTRACT

Noise is an important factor in image quality. We analyze it in images produced by digital cameras. We show that, beyond the usual standard deviation measurement, spatial correlations also convey interesting information which allows to (i) better describe the perception of the noise, (ii) analyze an unknown imaging chain. Indeed, knowledge of these spatial correlations is *necessary to predict the noise* after the rescaling and sampling involved in a realistic imaging chains.

1. INTRODUCTION

Noise is a crucial aspect of image quality. It is often measured by the standard deviation of the intensity level with respect to a true image (often a uniform patch, sometimes an edge or an oscillation). Standard practice are described in [6]. However a satisfactory characterization is still unavailable and therefore "the subject of ongoing research" as stated in the same document. This work is a contribution to this endeavor.

This usual measure of noise has a major drawback that knowing it at a given resolution does not allow a prediction of its level at other resolutions. We show that this standard framework must be supplemented by the **correlation function** of the noise if one is to be able to predict noise after the various treatments occurring in imaging chains before the actual printing or viewing of the image. We show how this function can be estimated. The correlation function in turn gives rise to a natural notion of noise size which we illustrate with natural images.

A perhaps unexpected application of the correlation function is the analysis of image processing occurring inside cameras. We give some real-life examples of this and an application showing how, because of these spatial correlations, it is a camera with *higher* standard deviation noise level that produces better prints. Finally we discuss how noise and blur (using our blur measure introduced in [4]) are balanced and use derived noise measures to optimize some image processing algorithms.

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2. A SIMPLE MODEL FOR NOISE IN DIGITAL CAMERAS

We assume that each (sensor) site is affected by an additive noise and that noises at different sites are independent on one another and on the "true" (or ideal) signal.[1] We do not assume that the law of the noise at a given site has a particular form (like Gaussian for thermal noise, or Poissonian for counting noise —see [2]). We furthermore assume that subsequent stages are *linear and translation invariant* (these stages include transfer of the charges within the CCD array, demoisacing, sharpening, etc.). Therefore the image produced by the camera is given by the convolution: $u = (u_0 + n_0) * K$ where u_0 is the ideal image at sensor level, n_0, the noise, is a random field with i.i.d. values at each site. The observed noise is $n := n_0 * K$.

Following standard practice, we shall call the standard deviation of the noise signal $\langle n(0)^2 \rangle$ the **noise level.** It is easy to compute this quantity from the kernel K:

$$\langle n(0)^2 \rangle = \left\langle \int n_0(x)K(-x)\,dx \int n_0(y)K(-y)\,dy \right\rangle =$$
$$\int \langle n_0(x)n_0(y)\rangle\, K(-x)K(-y)\,dxdy = \sigma_0^2 \int K(x)^2\,dx$$

using $\langle n_0(x)n_0(y)\rangle = \sigma_0^2\delta(x-y)$. Thus it is $\sigma_0^2\|K\|_{L^2}^2$, $\|K\|_{L^2}$ being the L^2 norm of the kernel.

An important consequence of this formula is how the noise level behaves under convolutions. Under a convolution by some kernel L, the standard deviation of noise becomes $\|K * L\|_{L^2} = \|\hat{K}\cdot\hat{L}\|_{L^2}$ using that the Fourier transform preserves L^2 norm and that that it maps convolutions into products. This formula implies that to predict the level of noise after an arbitrary convolution, one needs a complete knowledge of $|\hat{K}(\omega)|$ for all $\omega \in [0,\infty)^2$, i.e., one needs to know this kernel.

[1] This assumption means in particular that we assume the gain of the sensors to be fixed.

3. CONVOLUTION KERNEL AND CORRELATION FUNCTION

Motivated by the previous remark, we introduce a way of recovering the kernel K from an image produced by the camera. The key tool is the **correlation function**:

$$\mathrm{cor}_K(x) := \langle n(0)n(x)\rangle$$

Let us compute it from the kernel K:

$$\begin{aligned}\mathrm{cor}_K(x) &= \int \langle n_0(0-v)n_0(x-w)\rangle\, K(v)K(w)\,dvdw \\ &= \sigma_0^2 \int K(w-x)K(w)dw = \sigma_0^2 \tilde{K} * K(x)\end{aligned}$$

where $\tilde{K}(x) = K(-x)$. At the level of Fourier transforms,

$$\widehat{\mathrm{cor}}_K(\omega) = \sigma_0^2 |\hat{K}|^2(\omega)$$

This implies:

Proposition 1 *Assume that the kernel K is real, symmetric ($K(x,y) = -K(-x,y) = -K(x,-y)$), positive at 0 and real analytic. Then $\hat{K}(\omega) = \sqrt{\mathrm{cor}_K(\omega)}/\sigma_0$.*

Thus, one can reconstruct the kernel K describing the inner workings of the camera from the correlation function cor_K. This function is a well-known object in probability theory. It captures many of the spatial properties of random fields such as n. Indeed, in the special but fundamental Markov case, the probability law is completely determined by its correlation function [8].

4. ESTIMATING THE CORRELATION FUNCTION

We have reduced the determination of the kernel K to estimating the correlation function cor_w. We present a scheme for doing this from an image containing an almost uniform zone. We are going to estimate the correlation function on a window $\mathcal{W}$ of $(2W+1)\times(2W+1)$ pixels. The ideal value of the image at some (x,y) will be estimated by the average: $\bar{u}(x) = (\#\mathcal{A})^{-1}\sum_{y\in\mathcal{A}} u(x+y)$ where $\mathcal{A} = W\mathbb{Z}^2 \cap ([-SW,SW]^2 \setminus \mathcal{W})$. The proposed estimator is:

$$\begin{aligned}c_{W,S,N}(k,\ell) = \frac{1}{N^2}\sum_{i,j=1,\dots,N} &(u(iW,jW)-\bar{u}(iW,jW))\times \\ &\times (u(iW+k,jW+\ell)-\bar{u}(iW,jW))\end{aligned}$$

Let $E = (\#\mathcal{A})^{-1}\sum_{x\in\mathcal{A}} u_{11}x_1^2 + u_{22}x_2^2$, $Q(x) = u_{11}x_1^2 + u_{12}x_1x_2 + u_{22}x_2^2$ and

$$\bar{\Delta} = N^{-2}\sum_{i,j=1,\dots,N}(u_1+2iWu_{11}+jWu_{12}, u_2+2jWu_{22}+iWu_{12}).$$

A tedious but straightforward computation yields:

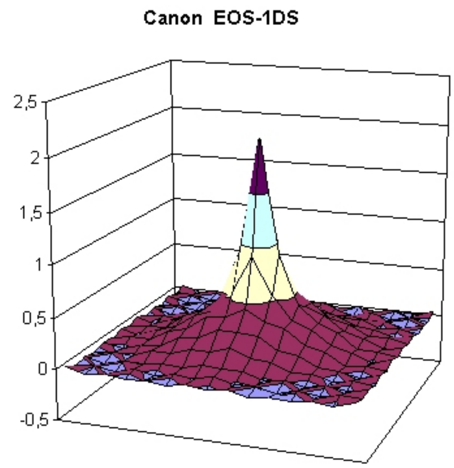

Fig. 1. Correlation function of the 11 megapixels Canon EOS 1DS at ISO 800 around grey level 128.

Lemma 1 *The expected value of $c_{W,S,N}(x)$ is:*

$$\mathrm{cor}_K(x) + \frac{\sigma_0^2}{(2K+1)^2-9} - 2E(Q(x)+x\cdot\bar{\Delta}) + E^2.$$

In particular, an unbiased estimator for $\sigma_0^2 = \mathrm{cor}_K(0)$ is:

$$\left(1 - \frac{1}{(2K+1)^2-10}\right)(c_{W,S,N}(0) - E^2).$$

The variance of these estimators are $\mathcal{O}(1/N^2)$.

We shall assume that due to small but unavoidable errors in the uniformity of lightning, the deterministic image is quadratic:

$$u(x) = u_* + u_1x_1 + u_2x_2 + u_{11}x_1^2 + u_{12}x_1x_2 + u_{22}x_2^2$$

We note that typical values given by a 5% vignetting from a 200 grey level across 500 pixels are $u_{11}, u_{12}, u_{22} \approx 4\cdot 10^{-5}$. Assuming $N = 100$, $W = 5$ and $S = 5$, the unestimated term above, $2E(Q(x)+\bar{\Delta}\cdot x) - E^2$, is about 10^{-3} which can be neglected by comparison with the root mean square error of the order of $1/N = 10^{-2}$. These estimates are compatible with the variability of measures actually performed by us.

5. FIRST APPLICATIONS

5.1. Analysis of cameras

We present the estimated correlation functions for two cameras in Figures 1-2. We observe the different shapes: circular and very peaked for the Canon EOS 1DS, more spread out and elongated in one direction for the other camera. A possible interpretation for this elongation is the leakage which can occur in the so called CCD-controller of these CMOS sensors.

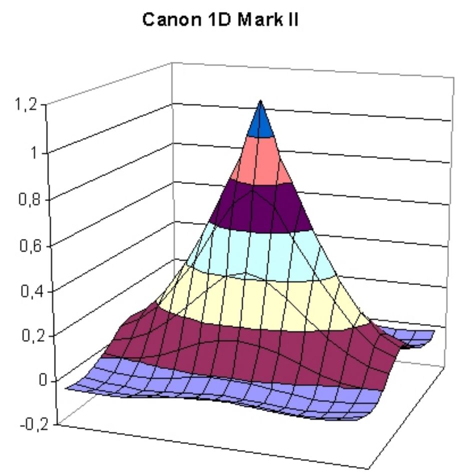

Fig. 2. Correlation function of the 8 megapixels Canon 1D Mark II at ISO 800 around grey level 128.

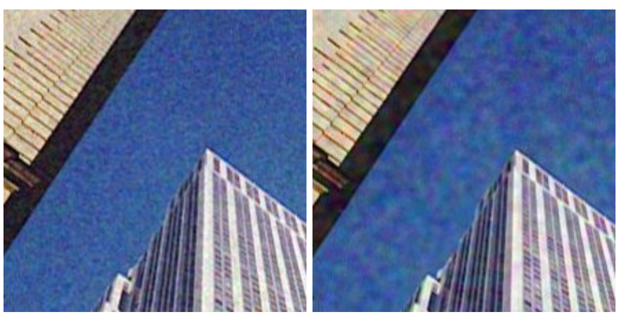

Fig. 3. A natural image with gaussian noise satisfying $\sigma^2 \approx 77$ and $\rho^2 = 1$ (left) or 8 (right).

5.2. Noise perception, standard deviation and spot size

We have performed a series of experiments to illustrate the roles of both the level of noise $\sigma = \langle n(0)^2 \rangle$ and the spread of the kernel ρ^2 ($\rho^2 = \int x^2 K(x)\,dx / \int K(x)\,dx$). See figures 3-4 which illustrate a same natural image to which has been added distinct gaussian noises with gaussian kernels. One sees that both σ^2 and ρ^2 are relevant to the perception of noise. The analysis of correlations between color channels can also be performed using a independent component analysis [5, 7] can be performed and will be presented elsewhere.

6. TRANSFORMATION OF NOISE IN IMAGING CHAINS

A zoom involves: a rescaling, an interpolation and a resampling (for a zoom-out the interpolation is more usually seen as an averaging but this is the same mathematically). The resampling has a trivial effect on the correlation function

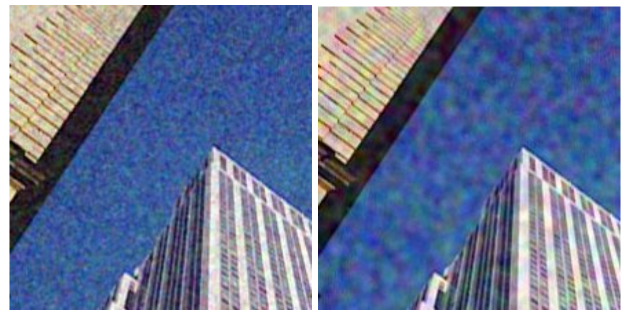

Fig. 4. A natural image with gaussian noise satisfying $\sigma^2 \approx 167$ and $\rho^2 = 1$ (left) or 8 (right).

(one just forgets its values outside of the designated lattice). A scaling by a factor λ (expansion of the size of the image by λ) just replaces the correlation function by $\mathrm{cor}_K(\lambda^{-1}x)$. The interesting part involves the interpolation. Assuming that it is linear, it is just a convolution with some kernel L. Thus, the zoom maps the image u to $(L * u)(\lambda^{-1}\cdot)$. The noise level changes from $\|K\|_{L^2}^2$ to $\|(K * L)(\lambda^{-1}\cdot)\|_{L^2}^2$. In general one cannot simplify this expression and computation of the new noise level requires the numerical evaluation of the underlying integrals.

To give a feeling for what is happening, let us consider the case where L and K are both Gaussian with $K = \frac{\sigma_0}{\pi\rho^2} e^{-x^2/(2\rho^2)}$ and $L = \frac{1}{2\pi\Delta^2} e^{-x^2/(2\Delta^2)}$. Then $L * K = \frac{\sigma_0}{2\pi(\rho^2+\Delta^2)} e^{-x^2/2(\rho^2+\Delta^2)}$. Therefore the level of noise after the zoom becomes:

$$\frac{\rho^2}{\rho^2 + \Delta^2}\sigma_0^2$$

That is, the averaging over a scale Δ larger than the typical correlation length ρ reduces the noise level, as could be expected.

Imagine now that one wants to produce 10×15cm print from the two cameras depicted in figures 1 and 2. A straightforward computation shows that in the first case, one will get a noise of $\sigma^2 \approx 0.2$. In the second case, one will get $\sigma^2 \approx 0.5$. Thus, even if initial σ^2 is higher for the first camera, as the noise is almost completely uncorrelated, it will decrease quickly as the image is zoomed out to produce the print. This illustrates the practical importance of the spatial correlations.

7. NOISE AND BLUR

Usually, improvement in sharpness comes at the price of increased noise and vice-versa. However this is not a general fact as the following computation shows. We use the *blur measure* called **BxU** developed in [4]. Under a convolution by K, this measure increases by $\mathrm{var}(K) = \int K(x)x^2\,dx / \int K(x)\,dx$.

We start with a discrete image u with impulsional noise σ_0^2 and make a convolution with the discrete kernel:

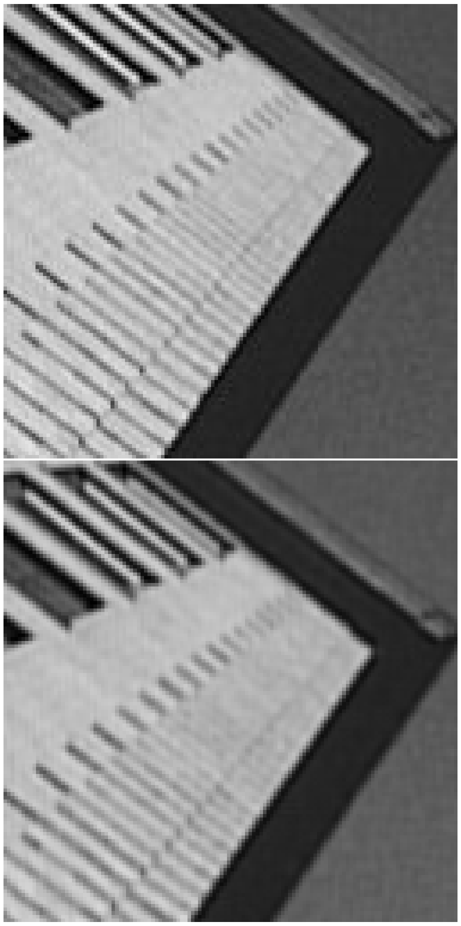

Fig. 5. A natural image and its "optimized" version.

$$K = \begin{pmatrix} c & b & c \\ b & a & b \\ c & b & c \end{pmatrix}$$

with a, b, c, some real numbers to be determined. The blur is increased by $\mathrm{var}(K) = 4b + 8c$. The noise level is multiplied by $\|K\|_{L^2}^2 = a^2 + 4b^2 + 4c^2$.

Now, we can for example chose a normalized K so that it does not increase blur and yet decreases noise level: minimize $a^2 + 4b^2 + 4c^2$ under the conditions $4b + 8c = 0$ and $a + 4b + 4c = 1$. We obtain:

$$\begin{pmatrix} -1/9 & 2/9 & -1/9 \\ 2/9 & 5/9 & 2/9 \\ -1/9 & 2/9 & -1/9 \end{pmatrix}$$

which nearly halves the noise (it multiplies it by $5/9$). The resulting image shows that such an operation (iterated four times) in fact creates *texture* through its complicated kernel. This is again an illustration of the importance of the correlation function beyond the usual noise level σ^2.

8. CONCLUSION

We have examined the standard measure of noise as the standard deviation of the difference to the ideal image. We have shown how an *a priori* uncorrelated noise at the sensor level in a digital camera acquires a non-trivial **correlation function**. This correlation function is necessary to estimate the noise level after further the image processing usually necessary to produce the desired prints or displays (where considering only the standard deviation can be grossly misleading). We have shown how to obtain this function from measurement on images produced by the camera. Experiments have further demonstrated the information gained on the inner workings of these cameras as well as the relevance of the correlation function to the perception of noise. Noises with the same standard deviation measure can be perceptually different if they have differing correlation functions. Finally we have shown how one can both lower blur and noise – the price, texturing, only showing up in the correlation function and not in the standard noise level.

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